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Non-linear Impact of Investments on Economic Growth

Abstract

In classical macromodels of economic growth the rate of GDP per one employee is a linear function of the investment rate. Meanwhile, strong social and organisational barriers appear, which slow down growth, with high investment rates and low consumption rates. That is why, we propose non-linear relations such as, for instance, the logistic function. A hypothesis about a non-linear impact of the investment rate on the rate of GDP per one employee will be verified on the example of the Polish economy in the years 1967–2001.

The relationship seems to be described best by the transformed logistic function. The results yielded by estimation of this function allow to presume that a high investment rate – above 30% leads to raising GDP growth rate by not more than seven percentage points (in comparison with the investment rate close to zero).

1. Labour productivity as a linear function of the investment rate

A typical tool in economic growth analyses is the two-factor production function with labour and fixed assets:

$$X_t = A_t f(L_t, K_t) \tag{1}$$

X_t – production in constant prices,

K_t – fixed assets in constant prices,

L_t – labour,

A_t – total factor productivity expressing technical, human capital or social capital level.

R. M. Solow (1967, p. 45) states that if the list of outlays is complete „*... then in line with the algebraic principle the growth rate of productive capacities is equal to the weighted sum of growth rates of outlays, with weights being elasticities of productive capacities in relation to each outlay*." In the model (1) the number of factors is, however, not complete. Assuming that each of production factors is paid for according to its final product and assuming the first degree homogeneity R. M. Solow proved that the following relation occurred:

$$\dot{X} = \dot{A} + (1-\beta)\dot{L} + \beta\dot{K} \tag{2}$$

where:

β – share of fixed assets in total production, $\beta \in (0;1)$

$1-\beta$ – share of labour in total production,

Dots denote growth rates (T. Tokarski 2001, pp. 23–24). Growth of A total productivity represents this fragment of economic growth, which cannot be attributed to factors of production introduced to function (2).

For the labour productivity function equation (2) assumes the following form:

$$\dot{X/L} = \dot{A} + \beta(\dot{K/L}) \tag{3}$$

where:

X/L – labour productivity,

K/L – capital labour ratio.

In the studies of economic growth conducted for group of countries equation (3) is frequently modified (see for example B. Liberda, A. Rogut, T. Tokarski 2002, pp. 400–402) to the form:

$$\dot{X/C} = \dot{A} + \beta\, I/X \tag{3'}$$

where:

C – number of population,

X – production (GDP) in constant prices.

In this model the growth rate of GDP per capita is dependent linearly upon the investment rate. Equation (3') can be derived from (3) on two assumptions:

1) the growth rate of capital labour ratio is equal to the investment rate,
2) labour grows at the same pace as population.

If the latter assumption is not fulfilled to an appropriate extent, the following model is more correct:

$$\dot{X} = \dot{A} + (1 - \beta)\dot{L} + \beta I / X \tag{2'}$$

in which GDP growth is dependent not only upon the investment rate but also on the labour growth rate or:

$$\dot{X} / L = \dot{A} + \beta I / X \tag{3''}$$

Productivity is estimated here per one employee and not one inhabitant.

The labour productivity growth rate in equations (3') and (3'') depends on the investment rate. According to C. Józefiak, an opposite dependence is equally strong. Labour productivity growth increases the utilisation degree of existing capacity and acts as an incentive for launching investments[1]. These dependencies should be described by a multivariate model (see: B. Liberda, A. Rogut, T. Tokarski 2002, pp. 397–409).

The OLS estimation results of linear model (3'') are following:

$$\dot{X} / L_{0-1} = \underset{(0.5)}{-1.24} + \underset{(2.6)}{13.1}\,(I / X_{0-1}) - \underset{(1.7)}{0.0068}\, inflation_{-1} + \underset{(2.1)}{0.12}\,(prog_{-2} - prog_{-4})$$

$$- \underset{(3.8)}{0.080}\, \dot{cr}_{0-1} - \underset{(9.9)}{9.3} u7982 \tag{4}$$

$$R^2 = 85.8\% \qquad DW = 1.65 \qquad S_e = 1.55$$

where:

X/L – labour productivity in Poland (GDP in constant prices per one employee), 1990=100,

[1] The talk took place on 16th September 2002. It can be added that labour productivity improves as a rule the company's financial situation, which is also an incentive for investing. A certain guideline for assessing the strength of these relationships is the length of GDP lag in the investment rate function. It appeared that the fit increased along with elongation of lags reaching even seven. We did not expect such a long lag. This outcome makes more probable a statement about bilateral interrelationships between the investment rate and the rate of GDP growth per one employee. Comparing the equations of investment rate and equation (4) it can be stated that the impact exerted by economic growth on investments dominates, whereas the impact of investments on economic growth is weaker and without any explicit lags (see: J.J. Sztaudynger 2003a). The same conclusion is reached by B. Liberda, A. Rogut, T. Tokarski on the basis of a survey of 29 countries and over 130 observations. They resemble also identical conclusions flowing from studies conducted by M. Blomstrom, R. E. Lipsey and M. Zajan published in 1996 (2002, p. 400 and p. 407).

$\dot{X}/L_{0-1}$ – labour productivity growth rate, mean value from present year and previous year in percentage points,

I/X_{0-1} – investment rate (decimal fraction), mean value from present year and previous year raised to the third power,

inflation – growth rate of prices of consumer goods and services in relation to previous year, in percentage points,

$\dot{cr}_{0-1}$ – growth rate of criminal offences confirmed by completed preparatory proceedings, mean value from present year and previous year,

prog – degree of privatisation of industry, work force in private sector in relation to total work force.

$prog_{-2} - prog_{-4}$ – increment of privatisation degree with a lag by two and by three years,

u7982 – dummy variable, 1 in the years 1979–1982, 0 in the remaining years.

The endogenous variable was defined as the mean value from the present period and the previous period in order to reduce fluctuations caused by agricultural production dependent heavily on weather conditions.

A insignificant constant term (rate of neutral technical progress) was obtained in model (4) with a linearly derived investment rate. It prompted a supposition that this relation can be non-linear.

2. Labour productivity as a polynomial function of the investment rate

C. Józefiak prompts an argument that strong social barriers slowing down growth appear at high investment rates and low consumption rates. There can appear also then organisational barriers of the so-called excessive investment programme observed in the 1970s. T. Tokarski, in turn, points out that the investment multiplier operates weakly with a low investment rate, and along with the growth of investment rate its operation intensifies[2]. In order to confirm this hypothesis polynomial functions and in point 3 the logistic function were used. Guided by the determination coefficient, significance of parameter estimators, value of a constant term assessment and striving to build a possibly simple model with a low degree of the polynomial, the third degree polynomial was considered to be the best one. It is presented in equation (5), which we estimated by the least squares method for the period 1967–2001:

[2] I conducted talks on this topic with C. Józefiak and T. Tokarski in September 2002.

$$\dot{X}/L_{0-1} = \underset{(2.6)}{2.20} + \underset{(3.5)}{25.2}\,(I/X_{0-1})^3 - \underset{(1.7)}{0.0065}\,inflation_{-1} + \underset{(2.2)}{0.12}\,(prog_{-2} - prog_{-4})$$

$$- \underset{(4.1)}{0.158}\,\dot{cr}_{0-1} - \underset{(9.9)}{8.9}\,u7982 \qquad (5)$$

$$R^2 = 87.5\% \qquad DW = 1.67 \qquad S_e = 1.46$$

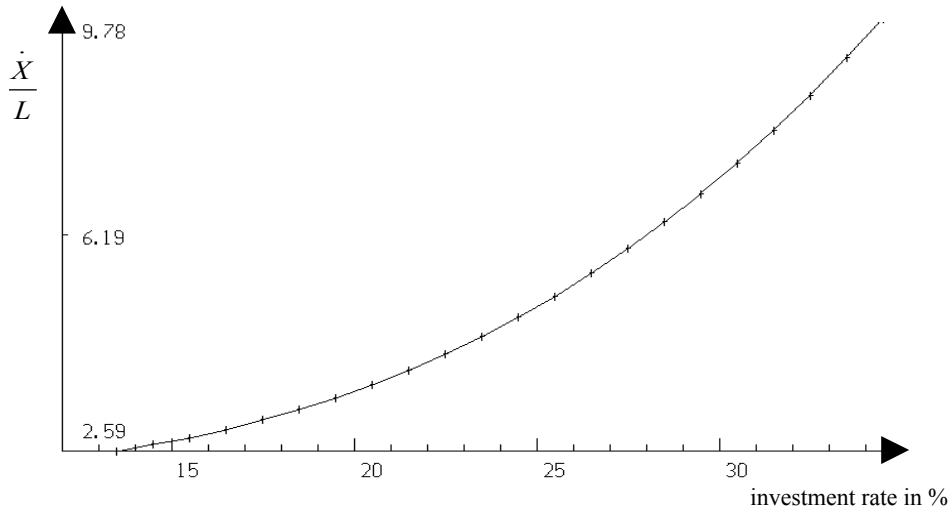
where:

$(I/X_{0-1})^3$ – investment rate (decimal fraction), mean value from present year and previous year raised to the third power, remaining denotations as in model (4).

Attempts to lag the investment rate were unsuccessful.

The assessment of neutral technical progress degree amounts to 2.2 (it is commonly believed that it is equal to about 2%). Figure 1 shows the shape of this function, which is, however, far from being perfect, as it 'explodes' at investment rates above 30%.

Figure 1. Labour productivity growth rate as a polynomial function of the investment rate $\dot{X}/L_{0-1} = 2.20 + 25.2\,(I/X_{0-1})^3$



Source: own estimations based on model (5).

It is claimed on the basis of mathematical economics that the growth path (of consumption) located highest is achieved at the investment rate of about one-third (T. Tokarski 2001, p. 131). Such high investment rates can be found sporadically, which makes it difficult to determine empirically the course of this function for large investment rates. It can be presumed that with the investment rate of about 30% the growth of the function presented in Figure 1 is slowed down. Therefore, we used a logistic curve, which is presented in the next point.

As it can be seen from Table 1 investments can be ascribed from one-eighth of labour productivity growth in 1993 to over nine-tenths growth in 1998 in the period between 1992 and 2001. It is on average almost a half (0.452) of growth achieved in this decade.

Table 1. Relation of investment growth to real labour productivity growth

Year	Relation
1992	0.414
1993	0.127
1994	0.157
1995	0.225
1996	0.331
1997	0.549
1998	0.918
1999	0.689
2000	0.465
2001	0.644
1992–2001	0.452

Source: own estimations based on the model (5).

If we accept the share of investments in labour productivity growth over the years 1992–2001 to be 1, then the share of privatisation will reach 0.63. On average 21 grosz from each zloty of GDP was allocated for investments in those years. In our opinion the costs of carrying out privatisation were many times smaller and its effects by only about one-third smaller.

3. Labour productivity as a logistic function of the investment rate

The hypothesis on nonlinear relationship between the labour productivity growth rate and the investment rate was confirmed in point 2. There was used a polynomial of the third degree, which allows to achieve much better economic and statistical results than the linear function. Its shortcoming is that it does not expire at high values of the investment rate. As it was said earlier strong social and organisational barriers slowing down growth appear at high investment rates. Hence, we are advancing a hypothesis that a dependence of the labour productivity growth rate on the investment rate has a logistic form.

Let us remind here of an analytical form of the logistic function:

$$y = a/(1 + b\, e^{-c\, z})$$

parameters a>0 b>0 c>0, whereas z accepts values from the interval (0, +∞). Due to the fact that the variable characterising the investment rate accepts values from the interval (0, d) and d is approximately equal to 0.37, we will use a function transforming the investment rate I/X from the interval (0, d) to its value from the interval (0, +∞)[3]:

$$z = (I/X)/[d - (I/X)]$$

where: d – maximum value that can be achieved by the investment rate.

Thus, it leads to the following modified logistic function:

$$y = a/(1 + b\, e^{-c\,(I/X)/[d-(I/X)]})$$

After replacing the linear relation in function (3") with the logistic dependence we obtain the following function of the labour productivity growth rate:

$$\dot{X}/L = \dot{A} + a/(1 + b\, e^{-c\,(I/X)/[d-(I/X)]}) \tag{6}$$

After supplementing the function with social capital represented by the crime rate, function (6) was estimated by means of the simplex method (see: W. Milo 1990, pp. 112–114) of non-linear estimation for Poland in the period 1967–2001. Moreover, there were introduced inflation and privatisation to the model similarly to point 2. Additionally, parameter d in the exponent at 0.365 was estimated before non-linear estimation. Results of this model's estimation are presented in equation (7) and equation (3) of previous article.

$$\dot{X}/L_{0-1} = 2.8 + 7.0/(1 + 21.5\, e^{-1.07*(I/X_{0-1})/(0.365-I/X_{0-1})}) \tag{7}$$

[3] This function was proposed by H. Klepacz.

where:

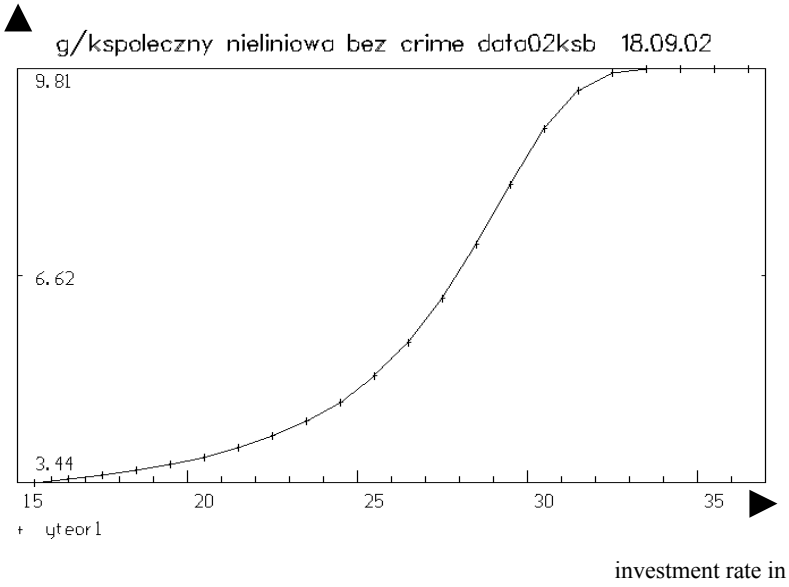
X/L – index of labour productivity in Poland (GDP at constant prices per one employee), 1990=100,

X/L_{0-1} – labour productivity growth rate, mean value from present year and previous year, in percentage points,

I/X_{0-1} – investment rate (decimal fraction), mean value from present year and previous year.

Interpretation of the estimation of parameter $b = 7.0$ is especially important from the economic point of view, as it shows by how many percentage points the economic growth rate can be raised maximally owing to investments. The course of function (7) is shown in Figure 2[4].

Figure 2. Labour productivity growth rate as a logistic function of the investment rate



Source: own estimates.

Marginal increments (7) and, in particular, the function's expiry at high investment rates are interesting from the economic point of view. These increments can provide a premise for decisions about not increasing the investment rate due to existence of social and technical-organisational barriers.

[4] An interesting question appears here concerning the analytical form of the function describing the labour productivity level.

B. Liberda, A. Rogut and T. Tokarski (2002) obtained for a group of 29 countries the estimate of 0.17 for the linearly derived investment rate, at which the neutral technical progress rate proved to be unessential.

Table 2. A comparison of marginal increments of labour productivity X/L in polynomial function (5) and logistic function (7)

Function	Third degree polynomial	Logistic
investment rate growth in % of GDP	marginal increment of labour productivity X/L in percentage points	marginal increment of labour productivity X/L in percentage points
from 14 to 15	0.06	0.03
15–16	0.07	0.03
16–17	0.08	0.03
17–18	0.09	0.04
18–19	0.10	0.04
19–20	0.11	0.05
20–21	0.12	0.06
21–22	0.14	0.07
22–23	0.15	0.09
23–24	0.16	0.11
24–25	0.18	0.14
25–26	0.19	0.18
26–27	0.21	0.24
27–28	0.23	0.32
28–29	0.24	0.44
29–30	0.26	0.61
30–31	0.28	0.82
31–32	0.30	0.97
32–33	0.32	0.86
33–34	0.34	0.40
34–35	0.36	0.05
35–36	0.38	0.00

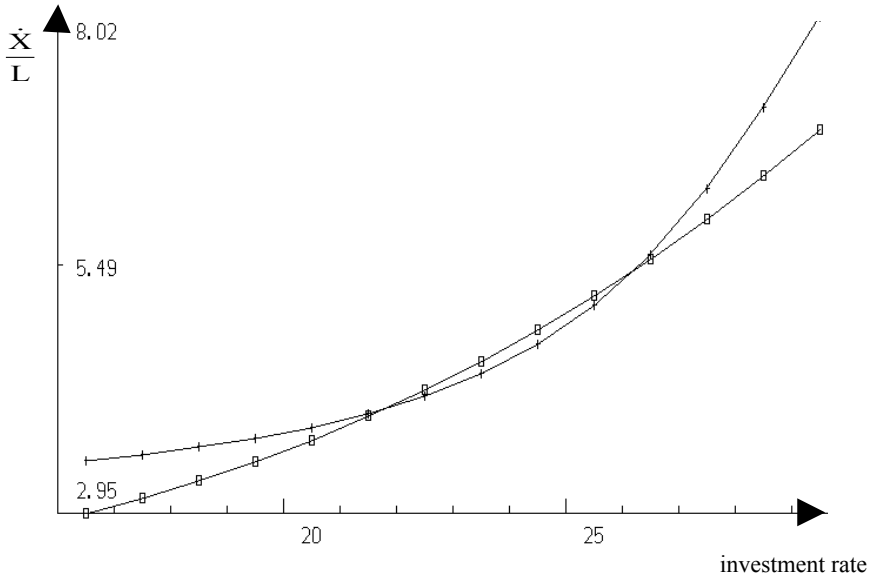
Source: own estimations.

We think that the results obtained for the third degree polynomial should be evaluated as doubtful due to too high values observed when investment rates exceed 35%. For the rates between 16 and 29% observed in the sample the

polynomial function is a certain approximation of the logistic function. It is illustrated by Figure (3):

The results of both functions in the interval 24–27% can be considered convergent with the 0.17 result obtained by B. Liberda, A. Rogut, T. Tokarski (2002)[5].

Figure 3. Labour productivity growth rates in logistic (7) and polynomial (5) functions of the investment rate



+ logistic function

▯ third-degree polynomial

Source: own estimations.

Low marginal increments of labour productivity for investment rates in the interval below 20% and, in particular, in the logistic function can be attributed both to a weak impact of the multiplier (on demand side) and the fact that 5–10% of investments can be of replacement type (on supply side).

[5] We never managed to obtain the convergence when estimating simultaneously in the logistic function all its four parameters. Therefore, two parameters were estimated interchangeably at one time, which was quite troublesome. Estimation difficulties and "similarity" in the function's course in the interval 20–27% of investment rates are in favor of using a simpler polynomial function.

It is argued basing on mathematical economics that the highest located growth path (consumption) is achieved at the investment rate of about 1/3 (Tokarski 2001, p. 131). A convergent result was obtained in the logistic function – at rates in the interval 31–33% the marginal labour productivity growth rate is close to unity and next it tends rapidly towards zero.

The constant term higher than in polynomial function (5) allows to notice that the more bent the function is in relation to low values of the investment rate the higher values it accepts.

4. Final remarks

We sought above a confirmation of the thesis that the relationship between the labour productivity growth rate and the investment rate is non-linear. This relationship seems to be described best by the transformed logistic function. The results yielded by estimation of this function allow to presume that a high investment rate – above 30% leads to raising GDP growth rate by not more than seven percentage points (in comparison with the investment rate close to zero). As it was shown in Table 2 the highest marginal labour productivity increments are obtained for investment rates in the interval 31–33%, in which these increments are close to unity. The inflexion point of the logistic curve is present for the investment rate equal to about 32% and marginal labour productivity increments diminish gradually. In our opinion, this result can be considered a confirmation of conclusions drawn on the basis of mathematical economics that the highest situated consumption growth path is attained at the investment rate of about 33%.

Such high investment rates seldom occur in the economy due to technical-organisational and social barriers (limited possibilities of lowering the consumption rate), which is to express the ‘expiry” of logistic curve. A question arises here whether the inflexion point of the logistic curve should not appear earlier, for instance, for the investment rate equal to 28–30%. Non-linear estimation procedures oriented specially at the estimation of logistic functions will allow perhaps to carry out a closer analysis of this problem.

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